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Dear Colleagues,

Please find below my review and recommendation for the habilitation degree of Dr. G. Oleksik.

Dr. G. Oleksik obtained a Master degree in the field of mathematics, specialising in the theoretical mathematics, at the Faculty of Mathematics and Computer Science, University of Lodz, in 2002. The title of his Master's thesis is "The Dvoretzky theorem" and the advisor was Prof. dr hab. Wojciech Banaszczyk.

After that he got his PhD, at the same university in November 2010, with the thesis "The Łojasiewicz Exponent of Non-degenerate Singularities," under the supervision of Professor dr. hab. Tadeusz Krasiński.

Since October 2022, Dr. G. Oleksik is an Assistant Professor at the Institute of Mathematics of the Poznan University of Technology.

Dr. G. Oleksik scientific achievements include 16 peer-reviewed publications in international scientific journals, 4 in conference proceedings, and several presentations at workshops and conferences, all his contributions adding quality and depth to his area of research.

The topics of Dr. G. Oleksik include singularity theory, specifically the study of the invariants of the germs of holomorphic functions near their singular points, i.e. complex power series with positive radius of convergence. We say that a holomorphic function has a singularity at a point if all its partial derivatives vanish at that point.

Studying and understanding singularities go back to I. Newton and it was consequently intensively developed thanks to the contributions of several major mathematicians such as J. Milnor, R. Thom, V.I. Arnold, amongst many others.

Several important topics are covered by Oleksik's research, one being the better understanding of the Milnor number and another one the famous Łojasiewicz inequality.

Many of Dr. Oleksik's results address, in particular, important questions from the Arnold's list of problems. For instance, which topological characteristics of a real (or complex) polynomial are computable from its Newton diagram?

The associated Newton polygon to a powers series, together with the family of its compact faces, constitute the Newton diagram of the series, a rather virtual

combinatorial object associated to a holomorphic function. Taking the polynomials corresponding to the compact faces of the Newton diagram, Kushnirenko (in the 1970s) introduced the concept of nondegeneracy, and under this condition, for an isolated singularity, Kushnirenko proves a famous formula for the Milnor number, an important topological invariant associated to an isolated singularity.

Many invariants, including the Milnor number, have been read from the Newton polyhedron of a generic singularity (i.e., singularity with generic coefficients).

Another important concept in the local singularity theory is the Łojasiewicz inequality for an isolated singularity. The relations between these concepts are the subject of Dr. Oleksik's research, which focuses on determining the discrete invariants of complex analytical singularities.

A well written detailed description can be found in his autobiographical report, where he presents six works on this topic, published between 2016 and 2023.

Notably his research concerns both the invariants of isolated singularities and the non-isolated singularities, and also the behaviour of the weighted-homogeneous polynomials at infinity.

In the sequel I will refer to the papers listed below:

1. Brzostowski, S., Krasiński, T., Oleksik, G.: The Łojasiewicz exponent of non-degenerate surface singularities. Canad. Math. Bull. 66: 1391 - 1410, (2023)
2. Oleksik, G.: The Łojasiewicz exponent of weighted homogeneous polynomials at infinity. Kyoto J. Math. 62: 403 – 415, (2022)
3. Eyral, Ch., Oleksik, G., Rożycki, A.: Lê numbers and Newton diagram. Adv. Math. 376: 1 – 21, 107441, (2021)
4. Brzostowski, S., Krasiński, T., Oleksik, G.: The Łojasiewicz exponent in non-degenerate deformations of surface singularities. Ann. Polon. Math. 127: 165 – 175, (2021)
5. Brzostowski, S., Oleksik, G.: On combinatorial criteria for non-degenerate singularities. Kodai Math. J. 39: 455 – 468, (2016)

In the paper 1), jointly with S. Brzostowski and T. Krasiński, they address the problem of computing the Łojasiewicz exponent of singularities, and for generic surface singularities f, they give an effective formula for in terms of the Newton polyhedron of f (a realisation of one of Arnold's postulates). This topic was intensively studied by famous mathematicians such as B. Teissier and M. Oka, for example.

In the paper 2) the author gives an explicit formula for the Łojasiewicz exponent at infinity for weighted homogeneous polynomials, assuming that the set of critical points is empty or one point, and the weights are greater or equal to 2.

Then, he gives a simple criterion for the properness property of the weighted homogeneous polynomial maps, assuming that the weights are positive. Notably, he proves that the Jacobian Conjecture is true in the pre-weighted case (this is a large

class of polynomials).

These are highly nontrivial results concerning the famous Jacobian Conjecture which is still wide open; any partial results are very important towards a better understanding of this very important conjecture.

In the paper 3) published in 2021 with C. Eyral, G. Oleksik, and A. Rózycki, in the top journal Advances in Mathematics, contains very interesting results concerning previously mentioned problems in the case of non-isolated singularities.

If the set of critical points is not isolated, i.e., has dimension $d > 0$, then the Milnor number is not well-defined. To address this situation D. Massey introduced a sequence of $d + 1$ numbers, called Lê numbers (the construction is topological, coordinate-dependent, but for a generic coordinate system, they are well-defined). The main result of the above work is the description of an algorithm that computes Lê numbers based on the Newton diagram, a very subtle combinatorial construction.

In particular, this implies the important fact that, if two non-degenerate singularities have the same Newton diagrams, then they have the same Lê numbers (if they exist), and their sets of critical points have the same dimension.

Together with Brzostowski and Krasiński, in the paper 4) the authors prove the constancy of the Łojasiewicz exponent for the non-degenerate μ-constant deformations of surface singularities, a partial affirmative answer to a question posed by B. Teissier.

In the article 5) , written jointly with S. Brzostowski, the authors demonstrate a crucial observation, namely that in Kushnirenko's key formula for the Milnor number, the assumptions can be weakened. That is, the convenience condition, i.e. that the Newton polyhedron intersects all coordinate hyperplanes, might not be necessary. In fact, the authors obtain a characterisation of the non-degenerate singularities by the combinatorial properties of the support and by the finiteness of the Newton number.

Dr. Oleksik has supervised numerous master and engineering theses in mathematics and data analysis. He has participated and has organised numerous scientific conferences. Amongst his organisational achievements, it is worth mentioning his work as the secretary (for 14 years) of the National Training Conference on Algebraic and Analytical Geometry, in Łódź.

Also it is worth noting his work promoting mathematics and cryptography at the the Marszałek S. Małachowski Secondary School in Płock, one of the oldest in Europe.

Dr. Oleksik has collaborators in Warsaw at IMPAN (Z. Jelonek, Ch. Eyral), in the Kielce University of Technology (A. Lenarcik, A. Płoski), and he continues to collaborate with Professor Krasiński's group in Łódź.

Moreover, he actively participates in the National Seminar on Singularity Theory.

Between 2008 and 2017, he was a researcher on two NCN grants, under the leadership of Prof. Spodzieja and Prof. Lenarcik respectively.

In 2019, the candidate received a National Science Center (NCN) Miniatura 2 grant, on the project "The Łojasiewicz exponent of real and complex polynomials", which he used to complete an internship at the renowned VIASM (Vietnam Institute of Advanced Study of Mathematics) in Hanoi.

In consequence, I consider Dr. Oleksik's scientific achievements very highly, thus in my opinion he fully deserves the degree of habilitated doctor in mathematical sciences.

Yours sincerely,

Laurentiu Paunescu